

When is $S=A/4$?

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Abstract

Black hole entropy and its relation to the horizon area are considered. More precisely, the conditions and specifications that are expected to be required for the assignment of entropy, and the consequences that these expectations have when applied to a black hole are explored. In particular, the following questions are addressed: When do we expect to assign an entropy?; when are entropy and area proportional? and, what is the nature of the horizon? It is concluded that our present understanding of black hole entropy is somewhat incomplete, and some of the relevant issues that should be addressed in pursuing these questions are pointed out.

Key words: Black Holes, Entropy, Quantum Gravity

Pacs: 04.70.-s, 04.70.Bw, 04.70.Dy.

Typeset using REVTeX

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I. INTRODUCTION

In the past 25 years there has been a great deal of activity around the nature and origin of black hole entropy, since the pioneer work of Bekenstein and Hawking who found a close relation between entropy and area of black holes [1–3]. It is generally regarded that the identification of the fundamental degrees of freedom and the computation of the entropy for a black hole is one of the major challenges for any candidate quantum theory of gravity. Of particular importance are the recent attempts to recover, from basic approaches to quantum gravity, the “standard expression” for the entropy of a black hole [4,5]. For a recent review see [6]. Our aim in this article is to review and discuss the foundations of those attempts by examining in detail under what conditions this standard answer can be expected to be obtained.

The question of when entropy is equal to $A/4$ for a black hole is in fact made of at least 3 questions:

1. To what exactly and under what conditions do we expect, in general, to assign an entropy?
2. When is the entropy assigned to a black hole equal to $1/4$ of its area?
3. To which area exactly do we refer in answering the above question?

The first (and probably the most controversial) of these questions is in itself a collection of questions:

1.a) Under what physical circumstances do we assign an entropy?

1.b) To what do we assign entropy?: To a history?, to an “instantaneous” state of a system?, to a our description of such a state of the system?

1.c) What needs to be specified in order to assign an entropy?

More concretely, question 1.a), namely under what circumstances we assign entropy, refers to the issue of assigning entropy to: i) all situations, ii) stationary situations, iii) quasi-stationary situations, or iv) some other set of situations, larger than ii) but smaller than i).

In answering this question, we are also led to the issue of what type of entropy are we talking about: thermodynamic entropy or statistical mechanical entropy. Within this later case we should also decide whether we refer to the Boltzmann or the Gibbs entropy.

Once one has decided when to assign entropy, one should also specify to what “object” it should be associated. For instance, one should say if the entropy is to be associated with the interior, the exterior or the horizon of a black hole. Finally, the question of what needs to be specified in order to assign entropy [(1.c)], refers to standard specifications that one provides in treating the statistical mechanics of any system, for instance, the distinction between system and observer, coarse graining, etc.

On a closer look, one can see that all the questions that make up the question of when to assign entropy, are in a sense, an indication that we need to put the concept of entropy on a firmer ground than it is at the moment. It might seem that the fact that statistical mechanics can deal successfully with all practical applications of the subject is an indication

that all is well and clear. However, to understand the limitations let us focus for a moment on another concept where the situation is fully clarified, but was not so when the concept was firstly used: The concept of energy. For a start, one realizes that for free particles the kinetic energy is conserved, and then, that for certain forces one can introduce a potential energy, where now it is the sum of the former and the latter that constitutes the true conserved energy. The story can be continued with the realization that one needs to incorporate more forms of energy in order to maintain the validity of the conservation law in more general circumstances. It is only when one regards energy as the “conserved quantity in the presence of time-invariance”, that the mystery behind all its apparent manifestations disappears, and one recognizes that one is truly talking in all cases about one single quantity: energy. Similarly, it seems that we would need an ‘unified’ definition of entropy similar to the one available for energy, in order to clarify the situation and in particular the discussion around question 1, namely the question of when to assign entropy to a system. In other words, in the case of energy one knows that one of the implied aspects of having a single energy conservation law, instead of, say, one for kinetic energy and one for potential energy, is the tacit acknowledgment that there exists physical process that convert one type of the energy into another. Moreover, given two types of energies, one can in principle (in almost all cases) find one physical process taking the first type into the second. The exceptions are in fact supposed to be codified through the concept of entropy and by the second law of thermodynamics (besides the usual limitations arising from other conservation laws). In the same way, if such a ‘unified recipe’ to deal with entropy was at hand, one would hope to understand not only the second law (in its generalized form including, in particular, the entropy of black holes), but also the limitations, if any, to converting one type of entropy into another.

This last point is particularly interesting, because as far as the authors know, no such restriction has been advanced to date and thus the second law is the only limitation for the conversion of energy from one type to another. In fact one can give strong arguments that such restrictions must exist, in particular, associated with the issue of locality: No one would believe that one is allowed to device a machine that, say, transfers heat from a cold reservoir to a hot one without any additional local effects, and which avoids violating the second law simply by having a second part of the machine that creates sufficient entropy in a distant galaxy. Such considerations clearly indicate that one must face the issue of localization of entropy in general, and by having a black hole as a part of such contraption, the localization of black hole entropy in particular. One would like to have a general definition, analogous say to the definition of the energy momentum tensor, of an entropy current S^a (see [6]) satisfying an equation of the form:

$$\nabla_a S^a \geq 0 \tag{1}$$

such that the entropy associated with an hypersurface Σ , with unit normal n^a , and volume element dV , given by,

$$\int_{\Sigma} n_a S^a dV, \tag{2}$$

be such that the entropy associated to an hypersurface is greater or equal to that associated with an hypersurface to the past of the first one.

Such a general and precise notion of entropy would seem to be required, before one can hope to have a complete understanding of the reasons behind the validity of a generalized second law (including the contribution to entropy associated with black holes). There are in fact proposals for the derivation of this generalized second law [7], which however fail to clarify the underlying reason for their validity, in the sense that, for example, the conservation of energy is understood as a result of a time invariance of the underlying theory. Needless to say, these issues can not be treated with the current understanding, because we lack, among other things, the notion of localization of the entropy.

Assuming that one is willing to consider entropy as assigned to all situations, one is then confronted with question 2: is the entropy associated to a black hole always proportional to its area? It is clear that in (quasi)stationary situations, the existence of the first law leads us to the conclusion that entropy *is* proportional to area. Then, the question narrows to: should we consider area as a measure of entropy, also for the dynamical case?

The third question then refers to the identity of the area horizon one wishes to consider in order to identify it with entropy. There are situations in which all known notions of horizon agree (if defined). In particular, this is true when the spacetime under consideration is stationary.

However, already in the quasi-stationary case there are differences between, say, the event horizon and an isolated horizon. In the dynamical case, none of the definitions agree. It is then of vital importance to “make up our mind” about the nature of the relevant horizon.

It is important to stress that the question of when entropy and area are proportional (question 2) and to the nature of the horizon (question 3) have to be considered once one has tried to answer question 1 in detail. In the hypothetical case that one has an “universal” definition for entropy, one might hope that question 3 would be settled ‘ab initio’ and that question 2 can be answered by a direct application of the relevant formalism (assuming one has a quantum theory of gravity).

The purpose of this work is to critically review our current understanding of the foundations of black hole entropy and to point out the interrelations (not always fully appreciated) between the positions one takes in answering the various questions here posed, and the requirements that the positions one adopts in facing each of these issues ought to be mutually consistent. It is important to stress that this article does not intend to give a global answer to the issues that are addressed, but rather to point out the unresolved issues.

In this work we restrict our attention to the general theory of relativity (with, in principle, arbitrary matter couplings), and do not consider higher derivative theories, for which the relation between entropy and area does not seem to hold even at the classical level (i.e., in the generalized first law) [8].

This paper is organized as follows: In Sec. II we discuss the question of when entropy is defined. Section III is devoted to the study of when entropy and horizon area are proportional. The question of the nature of the horizon is the subject of Sec. IV. Finally, we end with a discussion in Section V.

II. WHEN IS ENTROPY DEFINED?

First, let us elaborate on the question of what kind of entropy we should focus on, namely thermodynamical vs statistical.

We know that the thermodynamic entropy is associated with stationary and quasi-stationary situations. In the case of black holes, this is normally reflected in the existence of the ordinary first law, and the thermodynamic entropy would be the quantity appearing there. Nevertheless, we are in fact interested in the statistical mechanical entropy because, as one can argue, it is the most general kind of entropy since, for every situation in which the thermodynamical entropy is defined, so is the statistical mechanical entropy. In these cases the two essentially agree, but there are situations in which the (standard) thermodynamic entropy is not even defined. Furthermore, it is the statistical mechanical entropy the object that, in principle, can be calculated from the basic microscopic theory, which in the case of a black hole's entropy contribution would be the quantum theory of gravity¹.

Now, the two types of statistical entropy, namely Boltzmann and Gibbs are, in principle, conceptually different. The first, depends on the exact microstate of the system under consideration and is defined as the logarithm of the number of microstates being “macroscopically indistinguishable” from the given one. This set is said to represent the mesostate. That is, if N_i denotes the number of microstates making up the i^{th} mesostate, then the Boltzmann entropy is,

$$S = \log N_i \tag{3}$$

whenever the microstate finds itself within the i^{th} mesostate.

The second is an “ensemble functional”, rather than a function of the actual physical state of the system. Associated with the ensemble there is a probability density ρ on the space of *microstates*, and the Gibbs entropy is

$$\int dx \rho(x) \log(1/\rho(x)). \tag{4}$$

However, in practice we use an essentially identical coarse graining prescription to define the level of uncertainty that allows us to construct either the notion of mesostate for Boltzmann entropy or the ensemble for the Gibbs entropy. Since the statistical mechanical entropy is more general than the thermodynamical entropy, the former must be defined at least in stationary and quasi-stationary situations [ii) and iii) above]. We know of no criteria that would be appropriate to specify a more general situation (case iv) above) so our options for when the statistical entropy is defined seems to be restricted to just the set i) (i.e. always) or the same as the thermodynamical entropy (i.e. stationary and quasi-stationary cases).

¹It is also known that from purely dimensional reasons, one would need to introduce Planck's constant \hbar in order to identify entropy with area [1]. This expectation was fully realized in Hawking's semi-classical calculation [3], thus indicating that black hole entropy is quantum mechanical in nature.

We would like to further argue against the choice of having the statistical entropy defined only in the stationary and quasi-stationary cases. Making this choice would render the second law as practically useless to prevent, for instance, the construction of perpetual motion machines (i.e. consider a contraption that moves in such a way as to avoid passing through a situation for which entropy is defined).

Moreover, this option would seem to destroy the Markovian nature of a physical theory, namely the property that the predictive power of a physical theory is not increased by considering the past together with the present, as compared to the consideration of the present alone. That is, if the present corresponds to a situation in which entropy is not defined, we could not preclude a future situation with a given entropy S_1 , in terms of our knowledge of the present, but could, for example, preclude that situation if we use (together with the second law) the fact that in the past the (closed) system was in a situation characterized by an entropy $S_0 > S_1$. One could argue that the Markovian aspect should be associated with the whole physical theory and not with each physical law separately. However, in this case, the fact that the second law is the only law incorporating a particular arrow of time, it seems difficult to imagine that the other known physical laws could restore the Markovian nature to the situation at hand. All these considerations seem to take us to the position of accepting 1.a.i), i.e *The statistical entropy should be defined for all situations* (See also [9]).

Now, in providing this answer we are in fact providing also a partial answer to the question of which object should one assign an entropy to [1.b) above] in the sense that we are assigning an entropy to a physical situation with a notion of localization in time. The alternative of assigning an entropy to a complete spacetime or a history (as indicated for example in [10]) would seem to make the concept of entropy completely useless in the sense of adding predictive power, and in particular as a tool for ruling out perpetual motion machines. In a sense we are puzzled as to what would be the current view of the authors in [10] about the area increase theorem in classical general relativity and its resulting connection with the generalized second law.

Another, more moderate view, would be to assign an entropy with a situation localized in time but within the context of a spacetime or history. Here again there are potential problems if we require the history to be known to a larger extent that is possible from a description in terms of initial data -a situation that could occur if we let quantum events play a decisive role in the selection of the possible histories, as in the examples discussed in [11]- as we could render the concept of entropy, again, useless in ruling out perpetual motion machines. These problems would be avoided if we accept that there be no supplementary information in the history.

Thus, from the previous discussion we are lead to the conclusion that in order for the concept of entropy to be useful in the ways we expect it to be, *we must assign entropy under all circumstances and to instantaneous physical situations*.

The remaining aspect of question 1.b), namely to which object to assign entropy is in fact incorporated within the question of what needs to be specified in order to define entropy [question 1.c) above] which we address next. It is obvious that we must specify at least the physical system to which one is about to associate an entropy, and the coarse graining needed to define the macrostate (in the Boltzmann scheme) or the ensemble (in the Gibbs scheme). This leads to the conclusion that in fact we do not assign entropy to a physical situation,

but *to our description of the physical situation*. This view is consistent with the assignment of entropy to, say, de Sitter and Rindler horizons as in [12]. This is a consequence of the fact that there is, in principle, no natural specification of the coarse graining, if no specification is given of the experimental procedure used to prepare the system. Thus, entropy would seem to be a relative concept, with different physicists disagreeing about the value of the entropy of a specific system at a given “time”. This in itself is not so worrisome, after all, other concepts like energy or length suffer from the same relativism and are nevertheless very useful. However, this indicates that we must specify the observer with respect to which entropy is to be assigned. It is even possible that the two specifications, observer and coarse graining become intertwined, as for example in the case of a Black Hole, for which one can think that the prescription to disregard whatever occurs inside is in fact part of the coarse graining [11], or part of the specification of the observer (whom we can think is restricted to move in the outside thus having no access to any information about the inside). This must not be interpreted as in any way saying that the entropy of a black hole, is associated with the internal degrees of freedom since that would lead to various problems [7,13], but only as the statement that there is entropy to be assigned in this case because we are disregarding the inside. That is, we leave open the possibility that, for instance, the entropy might have to do with those degrees of freedom of the inside which can affect the outside through boundary conditions, or correlations, which would lead to the view that the relevant degrees of freedom are those associated with the horizon [13]. Returning to the issue raised in 1.c), in particular, to the details of the specification of observer and its observational capacity, we must, in accordance with the limitations imposed by the relativistic nature of physical reality, agree from the onset that the observer must be replaced by a collection of observers. We are faced then with the problem of having to specify the extent of this set of observers in order to be able to decide the extent of the observable quantities, a specification that one could hope will take into account, for example, the existence of horizons of various sorts. In particular, in the case one wished to consider event horizons in the previous discussion one would immediately run again into the problems derived from the teleological nature of such object, i.e. the fact that the present location of the horizon depends strongly on events in the future that might, as in the example considered in [11], make it impossible to predict its location based on the full knowledge of the system at present.

III. STATIONARY VS. DYNAMIC

Now we turn to question 2. That is, when is the entropy of a black hole proportional to its area? Here again we seem to face various alternatives: 2.a) In stationary and quasi-stationary situations; 2.b) Always; and 2.c) Some other restricted set of circumstances.

If we take the option that entropy and area are proportional only in stationary and quasi-stationary situations, we immediately face two questions. First, what are we going to take as the expression for entropy in other situations? One can try to answer this question both at the classical level and in the quantum domain. Classically, one would have to define a geometrical quantity to be associated with a dynamical entropy. Several such attempts are available in the literature [8,14,15]. Note that the particular prescription becomes intertwined with our question 3, that is, with the nature of the horizon one wishes to consider.

A second possibility is that the answer will be available only when we have a full theory of quantum gravity. After all, if asked to compute the entropy of a given non-equilibrium (macroscopic) configuration of a mass of gas, we need to go to the microscopic theory to count microstates etc, and we can not expect the answer to be a simple function of a single macroscopic parameter (which might not even be defined). Furthermore, within a full quantum theory of gravity, a generic configuration might not even have a macroscopic description in terms of a space-time with a horizon in it (like in some of the D-brane calculations [4]).

An important issue in both cases is, in a sense, the other side of the same coin, i.e., what are we going to make of the area theorem? If the dynamical entropy is not to be identified with the area of the event horizon, the existence of this theorem will lead to a strange situation because we will have on the one hand, the second law for the true entropy (which would not be a simple expression) and the second unrelated non-decreasing quantity: the area of the event horizon.

Furthermore, we note that the first law of black hole mechanics:

$$\delta M = \frac{\kappa}{8\pi} \delta A + \delta W \quad (5)$$

where M is the ADM mass, κ the surface gravity, A the area of the event horizon, and δW stands for work terms, is known to be valid for arbitrary variations of stationary black holes [16] even if these configurations are unstable. Thus, the point of the phase space to which the variation has taken the original stationary black hole, is not only not stationary, but it can not be said to be a configuration that would remain close to a stationary one, i.e. can not be said to be quasi-stationary. Nevertheless the identification of this law with the first law of thermodynamics, clearly indicates that we are assigning an entropy $S \sim A$ to these black holes.

Assuming that we take option 2.b), namely that the entropy is always proportional to the horizon area, then we are lead to a problematic situation because of the fact that we need to know the area of which horizon we are talking about. The event horizon seems not to be adequate since we need to know the complete spacetime in order to locate the horizon, and, as we concluded previously, the entropy need to be assigned to an instant of time, which in general relativistic settings corresponds to Cauchy hypersurfaces. We could take the view that the prescription is, then, to take the data associated with the hypersurface, including the geometry and the matter fields, evolve it according to Einstein's equations and proceed to locate the horizon in the corresponding hypersurface. Unfortunately this is also not a viable option in general as demonstrated by the example discussed in [11], in which the initial data, although complete, is not enough to locate the horizon on account of a decisive role played by a quantum measurement that is to be performed to the future of the given hypersurface, leading to fluctuations of the area of the event horizon with

$$\Delta A \approx A. \quad (6)$$

The previous discussion seems to lead us to option 2.c), that is, to some other restricted set of circumstances, which still needs to be specified. In this regard, again the analysis of the example in [11] would point to the following generalization of the previous prescription: take the data associated with the hypersurface, including the geometry and the matter fields,

consider the possible evolutions taking into account quantum alternatives and proceed to locate the horizon on the initial hypersurface for each of the corresponding spacetimes, and finally add the corresponding values of the areas with the appropriate probabilistic weights. So far we have centered our discussion of this question assuming that the entropy would be associated with the area of the event horizon, and in fact, the alternative 2.c) (some other restricted set of circumstances) is also an opening for the consideration of the next section.

IV. AREA OF WHAT?

In the past sections we have argued that the event horizon, even when it has a clear space-time definition, and is in a sense the obvious choice one might make, has several problems for a satisfactory definition of entropy. Again, the main problem of choosing the event horizon is its teleological nature that makes the situation *different* (as explained in Ref. [11]) from the case of an ordinary thermodynamical system put in a quantum superposition of states. As explained in [11], if one wishes to adopt the event horizon, then one needs to give up a canonical theory and/or modify the existing quantum theory. On the other hand, if one is not willing to give up a canonical quantum theory, then one can not consistently insist on the event horizon as the relevant quantity. One might conclude that, in this case, the event horizon should be replaced by another geometrical quantity in dynamical situations and then one is faced with the problem of finding a suitable alternative. The purpose of this section is to review the available possibilities, which to our knowledge are: 1) The apparent horizon [17], 2) The isolated horizon [18,19], and 3) The trapping horizon [15].

The apparent horizon has very serious problems, since it is known to be discontinuous for dynamical situations like a collapsing star [20], [11]. Furthermore, it is known that even the Schwarzschild spacetime contains Cauchy hypersurfaces with no apparent horizons.

The second alternative, namely isolated horizons, are particularly interesting for several reasons. First, it has been shown that for *quasi-stationary* processes, the (quasi-local) horizon mass satisfies a first law in which the entropy is proportional to the horizon area [18]. Secondly, there exists a calculation of the statistical mechanical entropy that recovers the “standard result” $S = A/4$ for various types of black holes [5]. This formalism is in fact a generalization of the standard stationary scenario to more physically realistic situations, because the exterior region need not to be in equilibrium. Nevertheless, the whole approach is based in the assumption that the horizon itself is in internal equilibrium. In particular, its area has to be constant, and nothing can “fall into the horizon”. In this regard, isolated horizons as presently understood, are not fully satisfactory since the formalism is not defined and does not work in general, dynamical, situations.

Moreover, there are situations in which one is faced with the occurrence of several isolated horizons, intersecting a single hypersurface, one within the other, and one must decide to single out the one to which entropy is to be assigned. We can take the view that this should be the outermost horizon, but this seems to be just an *add hoc* choice, unless it is argued that the selection is the natural one associated with the fact that we are specifying the “exterior” observers to be the ones with respect to which entropy is assigned. This view would be natural if we take the position that the assignment of entropy is related to the coarse graining, which is partially specified by pointing out the region from which

information is available to the observer. However, this point of view would conflict with the fact that the isolated horizons are not good indicators of such regions, basically because their definition is purely local and thus not fully based on causal relations.

Isolated horizons are well defined for equilibrium situations. If some matter or radiation falls into the horizon, the previously isolated horizon Δ_0 will cease to be isolated, and (one intuitively expects) there will be in the future a new isolated horizon Δ_1 , once the radiation has left and the system has reached equilibrium again. One would like to have a definition of horizon that interpolates between these two isolated horizons Δ_0 and Δ_1 , such that the physical situation can be described as a generalized horizon that “grows” whenever matter falls in. There is a natural direction for this notion of horizon, and this leads us to the third possibility, namely, trapping horizons [15].

In a series of papers, Hayward has been able to show that there exists (at least in the spherically symmetric case) a dynamical (as opposed to quasi-stationary) first law, for a (quasi-local) energy that, however, does not coincide with the horizon energy of the isolated horizons formalism (in the static limit). There exists also a second law, for the area of the trapping horizon, when a particular foliation of the space-like horizon is chosen. However, we face the problem that, by definition, these horizons can be specified only when the full spacetime is available: given a point in spacetime, the issue of whether or not it lies on a marginally trapped 2-surface, can not in general, be fully ascertained until the whole spacetime (where the rest of the 2-surface is to be located) is given. These option is also problematic because the trapping horizons are in general space-like and thus there is no guarantee that a given hypersurface would not intersect the horizon in several components thus leading to the same problem of in-definition that was mentioned in connection with option 2). Moreover, in this case the horizon can even be tangent to the hypersurface which is an extreme version of the previous problem. The fact that all this objections can be raised against this option, has its origin in the fact that the trapping horizon is not a surface defined on the grounds of causality alone.

It would be interesting to have a coherent formalism developed, which incorporates both the isolated and the trapping horizons formalisms, and that allows for a dynamical description of “black hole horizons”.

V. DISCUSSION AND CONCLUSIONS

In this paper, we have critically reviewed our current understanding of black hole entropy, focusing on the conceptual foundations that lie below the assignment of entropy to a black hole. In particular, we have argued that for the full content of the second law to be useful one needs to take the view that entropy should be assigned to all physical situations. Moreover we also argued that entropy should be assigned to situations localized in time, which in the general covariant setting implies that it should be assigned to Cauchy hypersurfaces, which can be viewed as immersed in spacetimes but only to the extent that the spacetimes themselves can be obtained from the data on the corresponding hypersurfaces.

The conclusions above seem to be very robust in the sense that taking an alternative viewpoint would seem to force us to deprive the second law from its predictive power and therefore from much of its meaning.

This has lead us to a rather paradoxical situation when dealing with the entropy of a black hole, because the alternatives that are available to play the role of entropy in the general dynamical case are all suffering from serious disadvantages: The event horizon can not in general be localized on a given hypersurface, and the best that can be expected is to have a probability for its various possible locations associated with the various possible spacetime developments of the given initial data. The apparent horizon would lead to the assignment of zero entropy even to some Cauchy hypersurfaces of the Schwarzschild spacetime. The Trapping horizons are also in general not localisable in the absence of the full spacetime and moreover are in general space-like, a feature that can lead to the multiple crossings of the horizon with the given space-like hypersurface, unless a preferred foliation of the horizon is chosen to begin with, and therefore a natural definition of “time evolution” along the horizon.

One possible conclusion is that the general expression for the entropy of a dynamical black hole that would arise from a complete quantum gravity theory, would be rather complicated and dependent on the details of the theory, a situation which would put such entropy in a similar footing with the entropies of other systems which, in the dynamical situation, are not expressible as simple functions of a few macroscopic parameters. The puzzling aspect of this view is the meaning we would ascribe to the classical area increase theorem, since we would have to abandon its interpretation as being just an expression of the second law, in the case of classical black holes.

Another possibility is that, in the semi-classical limit of the full theory, the states that yield space-times with a horizon in it, would have the property that the horizons are quasi-stationary (or even isolated). ‘Dynamical states’ would be present but they might not be interpreted as a classical space-time with a dynamical horizon. This drastic conclusion would of course have deep implications on black hole evaporation and information loss. This would imply that there are no quantum counterparts to classical dynamical black holes in certain regimes.

An alternative (an more moderate) conclusion would be to take the entropy as the probability-weighted average of the event horizon areas associated with the possible future developments of the appropriate Cauchy data. This was, for instance, the view taken in the analysis of [11] where an argument in favor of this proposal was obtained from a sum-over-histories formulation of quantum mechanics, together with the hypothesis that taking $S = -\text{tr}(\rho \ln \rho)$ with ρ the density matrix for the exterior black hole region yielded the correct result $A/4$ in the standard situations.

This discussion leads us to conclude that quite aside the issue of finding a correct theory of quantum gravity, the status and interpretation of the thermodynamics of black holes is rather incomplete. Furthermore, the basic issues raised in its connection could prove fundamental as guidance in the search for the quantum theory of gravity².

²Thus, in contrast with the stationary black hole situation, for which there seems to be no clues distinguishing the various approaches towards a theory of quantum gravity (as has been argued for instance in [21] (however see [22])), we expect the dynamical black hole case to be a much more demanding, therefore a more selective test for such proposals.

ACKNOWLEDGMENTS

We would like to thank A. Ashtekar for helpful comments. This work was in part supported by DGAPA-UNAM Grant No. IN121298 and by CONACyT grants J32754-E and 32272-E. AC was also partially supported by NSF grant No. PHY-0010061.

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